Postgraduate Education in Nonlinear Dynamical Systems and Automatic Control in Aerospace *

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Abstract: This note deals with the postgraduate educational programmes offered by the Department of Mathematical Modeling (BMSTU) both in English and in Russian. They include the Masters and PhD programmes in control of nonlinear dynamical systems. The programmes are specifically focused on nonlinear control techniques. The students are provided both with the theoretical courses in modern nonlinear control theory and applied courses in control of spacecrafts, aircrafts and mobile robots. Application examples in aerospace control are considered.

Keywords: Control education, Nonlinear control, Automatic control, Aerospace

1. INTRODUCTION

The last three decades have witnessed the great breakthrough in nonlinear control theory, see e.g. Kokotović and Arcak (2001). Nonlinear models and nonlinear techniques play now the central role in control engineering, since they allow to describe nonlocal behavior of a dynamical system and obtain results that are valid in large regions of the state space, see e.g. Isidori (1995), Khalil (2002) and Krstić et al. (1995).

However, to our knowledge, at the same time there is a serious lack of the postgraduate educational programmes properly covering the great variety of nonlinear design tools. One can find a lot of masters programmes in automatic control dealing with numerous applications and linear control theory. But, unfortunately, most of them don’t provide the comprehensive knowledge of nonlinear control techniques.

Meanwhile, the efficiency and performance of the nonlinear approach is often shown for the applications that are academic ones, see Fantoni and Lozano (2002). But even at the stage of education industrial applications of control theory are very important. Among such applications are aircrafts, helicopters, unmanned aerial vehicles, missiles, spacecrafts and spacestations. Nonlinear mathematical models and nonlinear control techniques allow to design globally or semiglobally stabilizing control laws and realize complex spatial maneuvers.

In this paper, we present our own viewpoint on nonlinear control education and propose the structure of postgraduate educational programmes in nonlinear control. The approach to education realized at BMSTU combines the profound knowledge of the nonlinear control methods with their applications to industrial systems in aerospace and other engineering disciplines. For instance, the following models of flying vehicles are considered as control objects: 6-DoF simplified model, 12-DoF model with real aerodynamic characteristics, 16-DOF model of helicopter and its simplified versions, 12-DoF model of quadrocopter and etc. There is a laboratory environment for the radio controlled helicopter available.

2. STRUCTURE OF THE POSTGRADUATE PROGRAMMES

At present there is no uniform understanding of the term "Postgraduate Education". Within the postgraduate educational programmes universities carry out both training for masters degrees and that for those who wish to pursue a PhD. Let’s consider both programmes simultaneously since they are closely interconnected.

BMSTU develops its own standards of the postgraduate education on the basis of the federal educational rules. That gives the opportunity to take into account modern directions and demands. The department of Mathematical Modeling offers postgraduate training in the area of nonlinear dynamical systems and automatic control. The point is to give the solid knowledge of the main nonlinear control techniques.

We propose the following structure of nonlinear control education.

Block 1. Differential geometric methods of system analysis and control design, see e.g. Isidori (1995), Krasnoschekchenko and Krishchenko (2005), Fliss et al. (1999), Chetverikov (2004), Sira-Ramirez and Agrawal (2004), and Krishchenko et al. (2002):
– state coordinates transformations and static feedback linearization;
– differentially flat systems and dynamic feedback linearization;
– relative degree and zero dynamics;
– control of nonminimum-phase systems.

**Block 2.** Stability theory and Lyapunov function design techniques, see e.g. Khalil (2002) and Sontag (2007):
– main Lyapunov stability theorems;
– Lyapunov function construction;
– input-to-state stability;
– control Lyapunov functions.

**Block 3.** Stability theory and Lyapunov function design techniques, see e.g. Khalil (2002) and Sontag (2007):
– dissipativity;
– storage functions;
– feedback passivation;
– cascade-connected designs.

**Block 4.** Integrator backstepping and forwarding, see e.g. Krstić et al. (1995), Kokotović and Arcak (2001):
– integrator backstepping designs;
– feedforward systems.

**Block 5.** Nonlinear state observers and output feedback, see e.g. Besançon (2007), Golubev et al. (2005), Khalil (2002) and Krstić et al. (1995):
– output injection observers;
– high-gain observer;
– observers for systems with monotonic nonlinearities;
– separation principle;
– observer-based backstepping.

**Block 6.** Nonlinear adaptive and robust control, see e.g. Krstić et al. (1995), Marino and Tomei (1995), Freeman and Kokotović (1996):
– robust control Lyapunov functions;
– robust integrator backstepping;
– adaptive integrator backstepping;
– model reference adaptive control.

The great attention is also paid to computer simulation and visualization using Matlab/Simulink, virtual laboratories and programming on C#.

The postgraduate programmes contain two possible tracks. The first is development of new results and trends in nonlinear control theory. The second direction has more applied character and includes control of flying vehicles and spacecrafts on the basis of their nonlinear models.

### 3. APPLICATION EXAMPLES

#### 3.1 Spacecraft control

Consider a spacecraft as the rigid body. Fix the body frame with the origin at the center of mass. It performs an angular rotation with respect to the inertial fixed coordinate system with the same origin. The position of the body-fixed frame with respect to the inertial reference frame at time $t$ is given by the quaternion

$$\mathbf{A}(t) = (\lambda_0(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)) \in \mathbb{R}^4$$

with the components normalized as follows:

$$|\mathbf{A}(t)|^2 = \lambda_0^2(t) + \lambda_1^2(t) + \lambda_2^2(t) + \lambda_3^2(t) = 1.$$ 

Angular rotation of a rigid body around its center of mass is described by the following system of kinematic and dynamic equations:

$$2\dot{\Lambda} = \Lambda \circ \omega,$$

$$I\dot{\omega} + \omega \times I\omega = u,$$

where $\omega = (\omega_1, \omega_2, \omega_3)^T \in \mathbb{R}^3$ is the vector of angular velocity projected onto the axes of the body-fixed coordinate system, $\circ$ stands for the multiplication of quaternions, $I$ is the inertia matrix of the spacecraft, and $u = (u_{11}, u_{22}, u_{33})^T \in \mathbb{R}^3$ is the control input. We assume that control is a continuous function of time.

The control problem is to rotate the spacecraft from the initial state

$$\mathbf{A}|_{t=0} = \Lambda_0 = (\lambda_{00}, \lambda_{10}, \lambda_{20}, \lambda_{30}),$$

$$\omega|_{t=0} = \omega_0 = (\omega_{10}, \omega_{20}, \omega_{30}),$$

$$u|_{t=0} = u_0,$$

to the given final state

$$\mathbf{A}|_{t=t_\ast} = \Lambda_\ast = (\lambda_{0\ast}, \lambda_{1\ast}, \lambda_{2\ast}, \lambda_{3\ast}),$$

$$\omega|_{t=t_\ast} = \omega_\ast = (0, 0, 0),$$

$$u|_{t=t_\ast} = 0.$$ 

To solve the control problem in question we construct the kinematic trajectory

$$\mathbf{A}(t) = (\lambda_0(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)), \quad t \in [0, t_\ast],$$

and the stabilizing feedback control.

The values of the functions $\lambda_i(t)$ and their first and second derivatives at the ends of the time interval $T = [0, t_\ast]$ are determined by the initial and final states of the system and the values of control in these states. Indeed, for $t = 0$ it follows from (1) that

$$\hat{\Lambda}(t)|_{t=0} = \hat{\Lambda}_0 = 0.5\Lambda_0 \circ \omega_0,$$

$$\ddot{\omega}_0 = (\dot{\omega}_{10}, \dot{\omega}_{20}, \dot{\omega}_{30})^T = \dot{\omega}(0) = I^{-1}(u_0 - \omega_0 \times I\omega_0).$$

At the same time, from (2) we have

$$\omega_0 = (\omega_{10}, \omega_{20}, \omega_{30})^T = \dot{\omega}(0) = I^{-1}(u_0 - \omega_0 \times I\omega_0).$$

Similarly, for $t = t_\ast$ we find the corresponding values $\hat{\Lambda}_\ast$, $\ddot{\omega}_\ast$ for boundary conditions.

Consider the polynomials $\mu_i(t)$ in $t$ of degree 5, satisfying these boundary conditions at $t = 0$ and $t = t_\ast$ for $\lambda_i$, and introduce the functions

$$\lambda_i(t) = \tilde{\mu}_i(t)/n(t), \quad i = 0, 1, 2, 3,$$

where $\tilde{\mu}_i(t) = \mu_i(t) + c_{i4}t^3(t - t_\ast)^3$, $c_{i4} = \text{const}$,

$$n(t) = \sqrt{\sum_{i=0}^{3} \tilde{\mu}_i^2(t)}.$$
Fig. 1. Plot of the programmed kinematic trajectory versus the time variable $\tau = t/t_\ast$, $\tau \in [0, 1]$

Functions (5) satisfy the same boundary conditions, see Ermoshina and Krishchenko (2000). Using these functions, we can obtain the desired programmed control:

$$
\begin{align*}
    u(t) &= 2I(\Lambda^{-1}(t) \circ \dot{\Lambda}(t)) \\
    &- \Lambda^{-1}(t) \circ \Lambda(t) \circ \Lambda^{-1}(t) \circ \dot{\Lambda}(t)) \\
    &+ 4\Lambda^{-1}(t) \circ \dot{\Lambda}(t) \times I_{\Lambda^{-1}(t)} \circ \dot{\Lambda}(t).
\end{align*}
$$

(6)

Constants $(c_0, c_1, c_2, c_3) = c_4$ determined by the optimization problem $J(c_4) \rightarrow \min$, where

$$
J(c_4) = \int_0^t \left( \frac{|u_1(t)|}{l_1} + \frac{|u_2(t)|}{l_2} + \frac{|u_3(t)|}{l_3} \right) dt,
$$

$l_i = \text{const.}$

The system (1)–(2) is affine in control, but it is not feedback linearizable. This follows, for example, from the fact that the function $|\Lambda|^2$ is a first integral of this system. The normalization condition $|\Lambda|^2 = 1$ determines a smooth 6-dimensional manifold $M = S^3 \times \mathbb{R}^3$ in the state space of the system. This manifold is invariant under system (1)–(2). Therefore, the restriction of this system to the manifold $M$ is well defined.

On the manifold $M$, we consider a smooth atlas of eight coordinate charts corresponding to the intersections of $M$ with the subspaces $\lambda_i > 0 \ (\lambda_i < 0)$ of the state space $\mathbb{R}^7 = \{(\Lambda, \omega)\}$. Writing out the restriction of the system (1)–(2) in local coordinates of these charts, one readily gets a feedback linearizable system. It turns out that all these systems can be obtained by restricting some affine system to the manifold $M$ considered as a submanifold in $\mathbb{R}^9 = \{(\Lambda, \dot{\Lambda})\}$ and given by the equations $|\Lambda|^2 = 1, \ d|\Lambda|/dt = 0$, see Ermoshina and Krishchenko (2000). It follows from (1)–(2) that this affine system coincides with the system

$$
\begin{align*}
    \ddot{\Lambda} &= \dot{\Lambda} - 2A \circ J^{-1}(\Lambda^{-1} \circ \dot{\Lambda}) \\
    &\times J(\Lambda^{-1} \circ \dot{\Lambda}) + \frac{1}{2} I \circ J^{-1} u.
\end{align*}
$$

Then, the feedback linearization technique can be used to find the stabilizing feedback control in local coordinates.

An example of angular maneuvers is shown in Fig. 1 – 3.

For the purpose of spacecraft control visualization, the special software package was developed, see Kavinov (2011).

Fig. 2. Plot of the angular velocities versus the time variable $\tau = t/t_\ast$, $\tau \in [0, 1]$

Fig. 3. Plot of the programmed control versus the time variable $\tau = t/t_\ast$, $\tau \in [0, 1]$

Fig. 4. Three-dimensional model of a spacestation

The software contains tools for visual design of three-dimensional models of spacecrafts and spacestations, see Fig. 4. The process of spacecraft motion and, in particular, the angular rotation under the action of the attitude control is simulated and visualized. The software package includes a set of predefined control laws and allows students to add their own ones.
3.2 Trajectory planning for an aircraft

Consider the problem of flying vehicle motion control under the following assumptions: 1) mass is constant; 2) there is no wind; 3) the earth surface is flat and non-rotating.

To describe the motion of the center of mass of a flying vehicle, we take the trajectory reference frame.

By allowing for the representation of the forces acting on the flying vehicle through the overloads and adding three differential equations relating the velocity vector with the spatial coordinates, we obtain the following system of six differential equations:

\[
\begin{align*}
\dot{V} &= (n_x - \sin \theta)g, \\
\dot{\theta} &= \frac{(n_y \cos \gamma - \cos \theta)g}{V}, \\
\dot{\psi} &= -\frac{n_y g \sin \gamma}{V \cos \theta}, \\
\hat{H} &= V \sin \theta, \\
\hat{L} &= V \cos \theta \cos \psi, \\
\hat{Z} &= -V \cos \theta \sin \psi,
\end{align*}
\]

where \( V \) is the velocity, \( m/\sec \); \( \theta \) is the flight path angle, \( \psi \) is the heading angle, \( \hat{r} \); \( \hat{H} \) is the altitude \( m \); \( L \) is the along-track deviation, \( m \); \( Z \) is the cross-track position, \( m \); \( n_x \) is the longitudinal overload; \( n_y \) is the transversal overload; \( \gamma \) is the roll angle, \( rad \); \( g \) is the sea-level acceleration of gravity, \( m/\sec^2 \).

The along-track position \( L \), altitude \( H \), and cross-track position \( Z \) are the coordinates \( x, y, z \) of the position of the flying vehicle center of mass in the normal earth-fixed reference frame. The overloads \( n_x, n_y \) and the roll angle \( \gamma \) are considered as the controls.

It is required to select a trajectory and corresponding controls such that moving along it the flying vehicle passes from the initial state

\[
x_0 = (V_0, \theta_0, \psi_0, H_0, L_0, Z_0)^T,
\]

to the given final state

\[
x_s = (V_s, \theta_s, \psi_s, H_s, L_s, Z_s)^T,
\]

which must be realized with the given precision:

\[
|\Delta x_i| = |x_i - x_{i+1}| < \Delta_i, \quad i = 1, \ldots, 6.
\]

The state variables must lie within the given ranges:

\[
V \in [V_{\text{min}}, V_{\text{max}}], \quad \theta \in \frac{\pi}{2}, \theta \in [\theta_{\text{min}}, \theta_{\text{max}}], \\
\psi \in \left[\psi_{\text{min}}, \psi_{\text{max}}\right], \quad H \in [H_{\text{min}}, H_{\text{max}}], \\
L \in [L_{\text{min}}, L_{\text{max}}], \quad Z \in [Z_{\text{min}}, Z_{\text{max}}].
\]

Similar constraints are also imposed on the controls:

\[
|\gamma| < \gamma_{\text{max}}, \\
n_{x_{\text{min}}} \leq n_x \leq n_{x_{\text{max}}}, \\
n_{y_{\text{min}}} \leq n_y \leq n_{y_{\text{max}}}.
\]

We also assume that in the initial and final states the values of controls

\[
\gamma_0, \quad n_{x_0}, \quad n_{y_0}, \quad \gamma_s, \quad n_{x_s}, \quad n_{y_s}
\]

and their tolerable deviations \( \Delta_{\gamma}, \Delta_x, \) and \( \Delta_y \) in the final state are known.

We introduce the following variables as the virtual controls for the system (7):

\[
v_1 = n_x, \quad v_2 = n_y \cos \gamma, \quad v_3 = n_y \sin \gamma.
\]

With these controls, (7) becomes an affine system of \( n = 6 \) equations with \( m = 3 \) control inputs:

\[
\begin{align*}
\dot{V} &= -g \sin \theta + v_1, \\
\dot{\theta} &= -\frac{\cos \theta}{g} + \frac{g}{v_2}, \\
\dot{\psi} &= -\frac{g}{V \cos \theta} v_3, \\
\hat{H} &= V \sin \theta, \\
\hat{L} &= V \cos \theta \cos \psi, \\
\hat{Z} &= -V \cos \theta \sin \psi.
\end{align*}
\]

System (14) has the form

\[
\ddot{y} = A(y, \dot{y}) + B(y, \dot{y})v,
\]

where

\[
y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad A(y, \dot{y}) = \begin{pmatrix} -g \\ 0 \\ 0 \end{pmatrix}, \\
B(y, \dot{y}) = \begin{pmatrix} \sin \theta & \cos \theta & 0 \\ \cos \theta & \sin \theta & 0 \\ -\sin \theta \cos \psi & -\sin \theta \sin \psi & \cos \psi \end{pmatrix}.
\]

The canonical state variables are

\[
y_1 = H, \quad y_2 = L, \quad y_3 = Z, \\
y_1 = V \sin \theta, \quad y_2 = V \cos \theta \cos \psi, \\
y_3 = -V \cos \theta \sin \psi.
\]

In the domain described by (10) the system (15) is solvable with respect to the controls

\[
v = B^{-1}(\dot{y} - A).
\]

Since the time interval is not defined, we take it equal to \([t_0, t_s]\) and determine the spatial trajectory \( \dot{H} = y_1(t), \)

\( \dot{L} = y_2(t), \quad \dot{Z} = y_3(t), \quad t \in [t_0, t_s], \)
satisfying all boundary conditions, that is, the given boundary conditions for state and control. To this end, we use relations (13) to calculate the boundary values of the virtual controls \( v(t_0) = v_0, \)

\( v(t_s) = v_*, \)

According to (15) and (16), the boundary conditions for state and the virtual controls at the ends of the time interval \([t_0, t_s]\) define the boundary conditions for the vector function \( y(t) \) and their first and second derivatives. Thus for \( t = t_0 \) we establish that

\[
y(t_0) = y_0, \quad \dot{y}(t_0) = \dot{y}_0, \quad \ddot{y}(t_0) = \ddot{y}_0.
\]

and for \( t = t_s \), similarly

\[
y(t_s) = y_*, \quad \dot{y}(t_s) = \dot{y}_*, \quad \ddot{y}(t_s) = \ddot{y}_*.
\]

Each of the components \( y_i(t), \ i = 1, 2, 3, \) of the smooth vector function \( y(t) \), satisfying the boundary conditions (18), (19) may be taken independently. For example, all of them may be found among the polynomials of the variable \( t \) of degree five. Indeed, let the boundary conditions

\[
f(t)|_{t=t_0} = f_0, \quad \dot{f}(t)|_{t=t_0} = \dot{f}_0, \quad \ddot{f}(t)|_{t=t_0} = \ddot{f}_0,
\]

\[
f(t)|_{t=t_s} = f_*, \quad \dot{f}(t)|_{t=t_s} = \dot{f}_*, \quad \ddot{f}(t)|_{t=t_s} = \ddot{f}_*
\]

be fixed for a smooth function \( f(t) \) defined over the interval \([t_0, t_s]\). We consider the polynomial of the fifth degree

\[
p(t) = \sum_{j=0}^{3} c_j (t-t_0)^j, \quad c_j (t-t_0)^j.
\]

For any values of the constants \( c_j \), the polynomial \( p(t) \) satisfies the boundary conditions (20) for \( t = t_0 \). For \( t = t_s \), the conditions (21) can always be satisfied by an appropriate choice of the constants \( c_j \). It is sufficient to substitute the polynomial \( p(t) \) into (21) and solve the resulting system of linear algebraic equations with respect to the unknowns \( c_j \).
To realize the above procedure for construction of the programmed control it is necessary to know the length \( t_n - t_0 \) of the time interval. However, this instant is not given in advance. The problem can be circumvented in part by passing to a new independent variable, see Krishchenko et al. (2009).

Let the programmed motion \((\bar{y}(t), \dot{\bar{y}}(t), \ddot{\bar{y}}(t)), \ t \geq t_0\) of the system (15) be synthesized. We design a continuously differentiable feedback control law \( v = v(y, \dot{y}, \ddot{y}) \) such that its values at the programmed trajectory coincide with the values of the corresponding programmed control

\[
v(\bar{y}(t), \dot{\bar{y}}(t), \ddot{\bar{y}}(t)) = \tilde{v}(t)
\]

and the closed-loop system (15) in the variables of the perturbed motion

\[
z_i = y_i - \bar{y}_i(t), \quad \dot{z}_i = \dot{y}_i - \dot{\bar{y}}_i(t), \quad i = 1, 2, 3, \tag{23}
\]

has the following form:

\[
\dot{z}_i + k_{i1}z_i + k_{i0}z_i = 0, \quad i = 1, 2, 3, \tag{24}
\]

where the constants \( k_{ij} \) are positive.

We notice that the matrix \( G(y, \dot{y}) = \frac{1}{g}B(y, \dot{y}) \) is orthogonal and, therefore, \( G^{-1} = G^T \).

The identity

\[
\ddot{\bar{y}}(t) = A(\bar{y}(t), \dot{\bar{y}}(t)) + gG(\bar{y}(t), \dot{\bar{y}}(t)) \tilde{v}(t) \tag{25}
\]

is valid for the programmed motion. By subtracting (25) from (15) one gets

\[
\dot{\bar{y}} - \ddot{\bar{y}}(t) = gG(y, \dot{y}) v - gG(\bar{y}(t), \dot{\bar{y}}(t)) \tilde{v}(t).
\]

Consequently,

\[
v = G^T(y, \dot{y}) G(\bar{y}(t), \dot{\bar{y}}(t)) \tilde{v}(t) + \frac{1}{g}G^T(y, \dot{y})(\ddot{\bar{y}}(t) - \ddot{\bar{y}}(t)). \tag{26}
\]

With allowance for (23) and (24), we finally obtain

\[
v = v(y, \dot{y}, t) = G^T(y, \dot{y}) G(\bar{y}(t), \dot{\bar{y}}(t)) \tilde{v}(t) + \frac{1}{g}G^T(y, \dot{y})(K_1(y - \dot{\bar{y}}(t)) + K_0(y - \ddot{\bar{y}}(t))),
\]

where \( K_1 = \text{diag}(k_{11}, k_{21}, k_{31}) \), \( K_0 = \text{diag}(k_{10}, k_{20}, k_{30}) \) are diagonal matrices. With this control, the programmed trajectory \( \bar{y}(t) \), \( \dot{\bar{y}}(t) \) of the closed-loop system (15) is globally asymptotically stable.

The vector function \( v = v(y, \dot{y}, t) \) defined by (26) is a set of auxiliary relations providing solution of the terminal control problem. The initial controls (longitudinal and transversal overloads and the roll angle) can be established from the virtual control using relations (13):

\[
n_x = v_1, \quad n_y = \sqrt{v_1^2 + v_2^2}, \quad \gamma = \arctan \frac{v_3}{v_2}. \tag{27}
\]

The established controls need not satisfy constraints (11). It is planned that in real fact the controls will be specified as follows:

\[
\tilde{n}_x = \text{sat}(n_x; n_{x, \min}, n_{x, \max}), \quad \tilde{n}_y = \text{sat}(n_y; n_{y, \min}, n_{y, \max}), \quad \tilde{\gamma} = \text{sat}(\gamma; \gamma_{\min}, \gamma_{\max}),
\]

where \( \text{sat}(x; a, b) = \min\{\max\{x, a\}, b\} \) is the saturation function.

Note that all calculations rely on the virtual controls \( v_1, v_2, \) and \( v_3 \). To take into consideration the constraints on the original controls, the current values of the virtual controls are recalculated into the main controls which are then corrected and recalculated back into the virtual controls.

Adjustment of the controls by the saturation function brings about an additional error in the result of motion modeling. This error can be so high that the motion trajectory does not reach the final point. Yet in some cases this distortion of the program controls can be eliminated using the stabilization mechanism so as the resulting trajectory is acceptable. Potential distortions in controls give rise to the need for additional testing of the determined trajectory. This testing is done by means of direct modeling of motion and analysis of its results.

Examples of programmed trajectories are shown in Fig. 5 and Fig. 6.

3.3 Virtual laboratory

For the control education purpose, the virtual laboratory software was developed to allow 3D visualization of control processes for various nonlinear systems, see Tkachev et al. (2012). It includes 3D models of inverted pendulum on a car, ball and beam system, reaction wheel pendulum, Furuta pendulum and models of the other most popular
Fig. 7. The virtual laboratory user interface: general view

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c1 = 0.1; c2 = 5/4; c3 = 1; c4 = 20; c5 = 20
v = (c5*c3*q2 + (c4 + c5)*q2_t - c1*c4*c5)*
math.tanh(c2*(c3*q1 + q1_t)) + c1*c2 * (c3*(c4 + c5)*
q1_t - ((c3 + c4 + c5)*q1 + c1_t)*q2_t)*2 +
g2*q2_t*math.cos(q2) + (c3 + c4 + c5)*g*math.sin(q2) +
2*c2*(c3*q1 - q1_t)*q2_t)**2 + g*math.sin(q2)**2
```

Fig. 8. The virtual laboratory user interface: control pane nonlinear systems. There is also the possibility to add new 3D virtual models. The virtual laboratory contains a set of built-in control laws such as feedback linearization, integrator backstepping and passivity based controls. It also allows to add user-defined control algorithms.

The overall view of the software user interface is shown in Fig. 7 and Fig. 8.

4. CONCLUSIONS

This paper suggests a structure of the postgraduate educational programmes in nonlinear control. The programmes are offered by the Department of Mathematical Modeling (BMSTU) both in English and in Russian and provide the solid knowledge of the main nonlinear control techniques. Applications in aerospace control and other control areas are demonstrated.

REFERENCES


